Caishan Liu, Peking University, China

Stability Analysis of a Bicycle Moving on a Surface

In this talk, we discuss the symmetry and stability of a bicycle, which is a typical multibody system subject to holonomic and nonholonomic constraints from the contact surface. In the case of the flat ground, we prove that the Lagrangian and the constraint distribution are invariant under the group action, satisfying the conditions of the principal kinematic case. The equilibrium points of the reduced system correspond to the straight line or circular motions of the bicycle. Noting that these equilibrium points are not hyperbolic, we analyze their Lyapunov stability according to the centre manifold theorem. For the case when the ground is a paraboloid of revolution, the principal kinematic conditions are not satisfied due to the complex constraints. We apply Lagrange's equation to establish an original DAE system, and use the rotation transformations to obtain a new DAE system, within which the time-dependent variables disappear. The study of the orbital stability of the original DAE system can then be transformed into the study of the Lyapunov stability of the new DAE system. Finally, we find the family of circular motions for the bicycle moving on a paraboloid of revolution, and analyze their stability in terms of the centre manifold theorem.

Caishan Liu is a Professor of Mechanics at Peking University. After obtaining his Ph.D. in 1997, he came to Peking University first as a Postdoc and then as a Professor in the Department of Mechanics. Since 2008, he has been a Professor in the Department of Aerospace Engineering. His research interests include dynamics of nonholonomic systems, physical mechanisms underlying contact, impact and friction, granular materials, and applications of mechanics to various aspects of engineering.