Demeter Krupka, Lepage Research Institute, Slovakia

Variational Forces

In this talk, the problem of extension of variational principles in classical mechanics to mechanical systems subject to forces depending on velocities is considered. As a basic tool, the Helmholtz equations of the inverse variational problem are applied. The aim is twofold. First, the basic assertions of the inverse problem theory for systems of second-order ordinary differential equations are summarized. Second, we present new results about the problem of variational forces. We consider the Euler–Lagrange expressions of general first-order Lagrangians $\mathcal{L}(t, x^j, \dot{x}^j)$. Assigning to $\mathcal{L}$ its kinetic energy component $\mathcal{T} = \frac{1}{2}(\partial^2\mathcal{L}/\partial\dot{x}^i\partial\dot{x}^j)\dot{x}^i\dot{x}^j$ yields the canonical decomposition $\mathcal{L} = \mathcal{T} - \mathcal{U}$. On the basis of this decomposition, we introduce variational forces, generalizing conservative forces of classical mechanics. For a restricted class of Lagrangians, $\mathcal{U}$ depends on $(t, x^j, \dot{x}^j)$, and we give a complete classification of $\mathcal{L}$. The theory works on tangent bundles of smooth manifolds; in particular, the Lagrangians admitting first-order forces are closely related to Finsler geometry.

Demeter Krupka received his Ph.D. in Geometry and Topology from Charles University in Prague and Dr.Sc. (Doctor of Science) from the Czechoslovak Academy of Sciences. His research was supervised by Andrzej Trautman (Warsaw, Poland) and Ivan Kolar (Brno, Czechia). He was appointed at Brno University, his Alma Mater, and other Czech and Slovak universities and, as a visiting professor, at numerous foreign universities (LaTrobe, Melbourne (Australia), Bahia Blanca (Argentina), BIT (Beijing, China), Torino and Florence (Italy), and many others). He also served as the rector of the Silesian University, Opava (Czechia). At present, he is the head of an international non-profit organization Lepage Research Institute (office at Presov University, Slovakia). His research is focused on global variational analysis, differential geometry, tensor algebra, and mathematical physics (http://www.lepageri.eu/); main results include foundations of global variational geometry, Finsler–Kawaguchi geometry, invariant variational problems on smooth manifolds, variational sequence and the inverse problem of the calculus of variations, differential invariants, natural variational principles, and the trace decomposition problem. He belongs to the founding editors of the Journal of Geometry and Physics, Differential Geometry and its Applications (Editor-in-Chief), and International Journal of Geometric Methods in Modern Physics. He was the Editorial Board member of Mathematica Bohemica, Mathematica Slovaca, Archivum Mathematicum (Editor in Chief), and others. At present, he is an editor of the book series Atlantis Series in Variational Geometry (Atlantis/Springer).