



# 北京理工大学

## 数学与统计学院学术报告

### Energy asymptotics of a nonlocal Brézis-Nirenberg problem involving critical Hartree equation

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时间：2026.05.10. 10:30-11:30

地点：中关村校区研楼 209

摘要：In this talk I will introduce a nonlocal version of the famous Brézis-Nirenberg problem related to the critical Hartree equation. In fact we consider the minimizing problem

$$S(\varepsilon V) = \inf_{u \in H_0^1(\Omega) \setminus \{0\}} \frac{\int_{\Omega} |\nabla u|^2 dx + \varepsilon \int_{\Omega} V |u|^2 dx}{\left( \int_{\Omega} \int_{\Omega} \frac{|u(x)|^{2^*} |u(y)|^{2^*}}{|x-y|^{\mu}} dx dy \right)^{\frac{1}{2^*}}},$$

where  $\Omega$  is a bounded open subset of  $\mathbb{R}^N$ ,  $N \geq 4$ ,  $0 < \mu < 4$  and  $2^* = \frac{2N-\mu}{N-2}$  is the critical exponent in the sense of the Hardy-Littlewood-Sobolev inequality. Under certain assumptions on  $V$ , we obtain the asymptotics of  $S(\varepsilon V) - S(0)$  to leading order as  $\varepsilon \rightarrow 0^+$  and give a precise description of the blow-up profile of minimizing sequences. Moreover, we characterize the concentration points as being extrema of a quotient involving the Robin function. If  $N = 3$ , we study the asymptotics of  $S(Q + \varepsilon V) - S(Q)$  to leading order as  $\varepsilon \rightarrow 0^+$  for the following problem

$$S(Q + \varepsilon V) = \inf_{u \in H_0^1(\Omega) \setminus \{0\}} \frac{\int_{\Omega} |\nabla u|^2 dx + \int_{\Omega} (Q + \varepsilon V) |u|^2 dx}{\left( \int_{\Omega} \int_{\Omega} \frac{|u(x)|^{2^*} |u(y)|^{2^*}}{|x-y|^{\mu}} dx dy \right)^{\frac{1}{2^*}}},$$

for  $0 < \mu < 2$  and further give a precise description of the blow-up profile of minimizing sequences under certain assumptions on  $Q$  and  $V$ . In particular, minimizers concentrate at a point in the zero set of the Robin function.

**个人简介：**杨敏波, 浙江师范大学数学科学学院院长, 教授, 博士生导师, 主要关注与研究非局域椭圆型偏微分方程(组)解的存在性、渐近行为与集中性质; 非局域临界热方程与Hartree方程解的稳定性与爆破分析; 非局域Lane-Emden方程及其相关问题解的分类; 具有强不定变分结构的微分方程解的存在性与集中性; 拟线性薛定谔方程的变分与分歧方法。在J. Lond. Math. Soc., Calc. Var. Partial Differential Equations, Mathematische Zeitschrift, Commun. Anal. Geom., J. Differential Equations, Nonlinearity, Commun. Contemp. Math., Pacific Journal of Mathematics, 中国科学(中、英)等国内外高水平学术期刊上录用发表论文130余篇。目前已主持国家自然科学基金面上项目、国际(地区)合作交流项目、省基金重点项目多项。