Dynamic Programming for General Linear Quadratic Optimal Stochastic Control with Random Coefficients

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We are concerned with the linear-quadratic optimal stochastic control problem where all the coefficients of the control system and the running weighting matrices in the cost functional are allowed to be predictable (but essentially bounded) processes and the terminal state-weighting matrix in the cost functional is allowed to be random. Under suitable conditions, we prove that the value field $\mathbb{V}(t,x,\omega),(t,x,\omega) \in [0,T]\times R^n \times \Omega$, is quadratic in x, and has the following form: $\mathbb{V}(t,x)=\langle \mathbb{K}_t x, x\rangle$ where $\mathbb{K}$ is an essentially bounded nonnegative symmetric matrix-valued adapted processes. Using the dynamic programming principle (DPP), we prove that $\mathbb{K}$ is a continuous semi-martingale of the form

$$\mathbb{K}_t = \mathbb{K}_0 - \int_0^t dk_s + \int_0^t \sum_{i=1}^d L_s^i \, dW_s^i, \quad t \in [0,T]$$

with k being a continuous process of bounded variation and the stochastic integral $\int_0^\cdot \sum_{i=1}^d L_s^i \, dW_s^i$ being a BMO martingale; and that $(\mathbb{K}, L)$ with $L := L^1, \cdots, L^d)$ is a solution to the associated backward stochastic Riccati equation (BSRE), whose generator is highly nonlinear in the unknown pair of processes. The uniqueness is also proved via a localized completion of squares in a self-contained manner for a general BSRE. The existence and uniqueness of adapted solution to a general BSRE was initially proposed by the French mathematician J. M. Bismut [in *SIAM J. Control & Optim.*, 14(1976), pp. 419–444, and in *Séminaire de Probabilités* XII, Lecture Notes in Math. 649, C. Dellacherie, P. A. Meyer, and M. Weil, eds., Springer-Verlag, Berlin, 1978, pp. 180–264], and subsequently listed by Peng [in *Control of Distributed Parameter and Stochastic Systems (Hangzhou, 1998)*, S. Chen, et al., eds., Kluwer Academic Publishers, Boston, 1999, pp. 265–273] as an open problem for backward stochastic differential equations. It had remained to be open until a general solution by the author [in *SIAM J. Control & Optim.*, 42(2003), pp. 53–75] via the stochastic maximum principle with a viewpoint of stochastic flow for the associated stochastic Hamiltonian system. The present talk introduces the *second but more comprehensive* (seemingly much simpler, but appealing to the advanced tool of Doob-Meyer's decomposition theorem, in addition to the DDP) adapted solution to a general BSRE via the DDP. The detailed full paper is referred to SIAM J. Control Optimization. 53(2), 2015.