Inverse problems for linear hyperbolic equations using mixed formulations

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We explore a direct method allowing to solve numerically inverse type problems for hyperbolic type equations. We first consider the reconstruction of the full solution of the wave equation posed in $\Omega \times (0, T)$ - Ω a bounded subset of $\mathbb{R}^N$ - from a partial distributed observation. We employ a least-squares technic and minimize the L^2-norm of the distance from the observation to any solution. Taking the hyperbolic equation as the main constraint of the problem, the optimality conditions are reduced to a mixed formulation involving both the state to reconstruct and a Lagrange multiplier. Under usual geometric optic conditions, we show the well-posedness of this mixed formulation (in particular the inf-sup condition) and then introduce a numerical approximation based on space-time finite elements discretization. We show the strong convergence of the approximation and then discussed several examples for $N = 1$ and $N = 2$. The reconstruction of both the state and the source term is also discussed, as well as the boundary case as well as the parabolic case. Joint works with Nicolae Cîndea. Details can be found in [1, 2] using arguments developed in [3, 4].

References

[1] N. Cîndea, and A. Münch, *Inverse problems for linear hyperbolic equations using mixed formulations.* To appear in Inverse Problems. http://arxiv.org/abs/1502.00114

[2] N. Cîndea, and A. Münch, *Reconstruction of the solution and the source of hyperbolic equations from boundary measurements: mixed formulations.* Submitted. (2015).

[3] C. Castro, N. Cîndea, and A. Münch, *Controllability of the linear one-dimensional wave equation with inner moving forces*, SIAM J. Control Optim., 52 (2014), pp. 4027–4056.

[4] N. Cîndea and A. Münch, *A mixed formulation for the direct approximation of the control of minimal L^2-norm for linear type wave equations*, Calcolo, 52 (2), 2015, pp. 1–4.